Trade-offs between robustness and small-world effect in complex networks

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Overview

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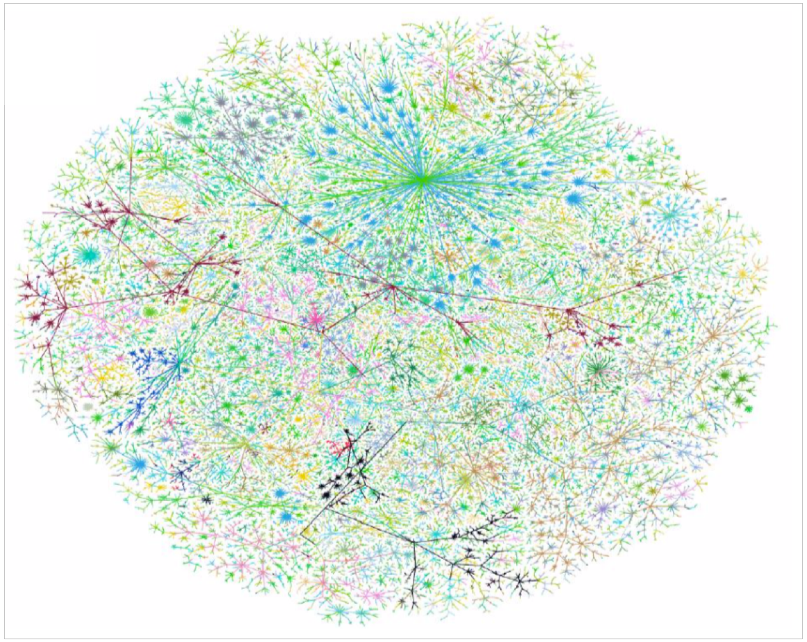
Introduction

Complex systems and networks

- Many real complex systems can be modeled as networks
- Function and behavior of networked systems can be largely influenced by their structural features
- Robustnesss and small-world effect are two crucial features which have attracted increasing attention



A visualization of the Internet at the level of »autnomous systems» , local groups of computers each representing hundred of thousands of machines.

Robustness (1/2)

- Ability of a network to maintain its connectivity when a fraction of nodes (links) is damaged
- Growing attention in many fields (ecology, biology, economics, engineering…)
- Real networks are results of complex processes and designing them from scratch is pratically impossible

↓

- Great interest in improving existing networks modifying the topology:
  - Adding links
  - Deleting links
  - Rewiring links

Robustness (2/2)

- Natural connectivity $\bar{\lambda}$
- Changes strictly monotonically with the addition or deletion of edges
- Mathematically can be derived from the graph spectrum $\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ as an average eigenvalue

$$\bar{\lambda} = \ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{\lambda_i}\right)$$

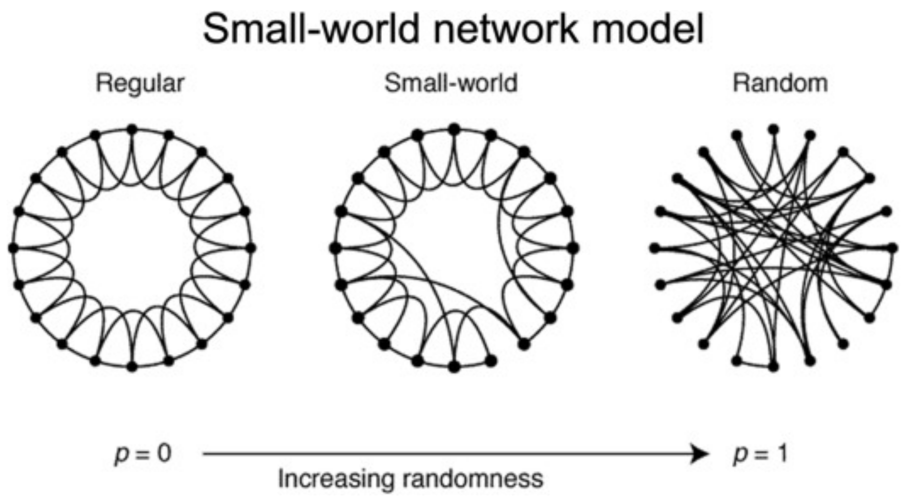
- Strong discrimination in measuring robustness and low computational complexity

Small-world effect (1/2)

- Most pairs of nodes are connected by a relatively short path through the network
- Distance d increases "slowly" with the number of nodes N

$$d \approx \log N$$

- Several implications: diffusion processes, cost-effectiveness analysis…



Small-world effect (2/2)

- Extent of small-world effect measured with efficiency (reciprocal harmonic average of shortest distance)

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}}$$

- Has some desirable mathematical properties:
  - Normalized to a range of $[0, 1]$
  - Valid for disconnected networks

Motivation

- Little has been done on joint optimization of robustness and other structural features
- Previous works focused on:
  - How the selection for robustness or small world effect influence topology [Netotea, Pongor, *Cellular Immunology,* 2013]
  - A tradeoff between small world effect and dynamical resilience [Brede et al., *Physics Letters,* 2006]
- These works did not preserve **node degrees**. For pratical purposes, changing the degree of a node can be more expensive then changing the connection

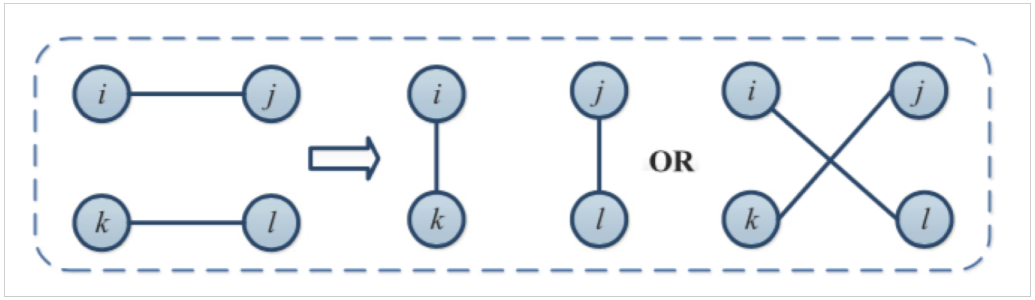
Scope

1. Demonstrate that there is a **conflinct relation** between robustness and small world effect for a given degree sequence
2. Propose a **multi-objective** trade-off optimization model
3. Develop a **heuristic algorithm** to obtain the optimal trade-off topology for both structural properties
4. Show that the optimal network topology exhibits a pronounced **core-periphery** structure

Robustness and small world effect: a conflicting relation

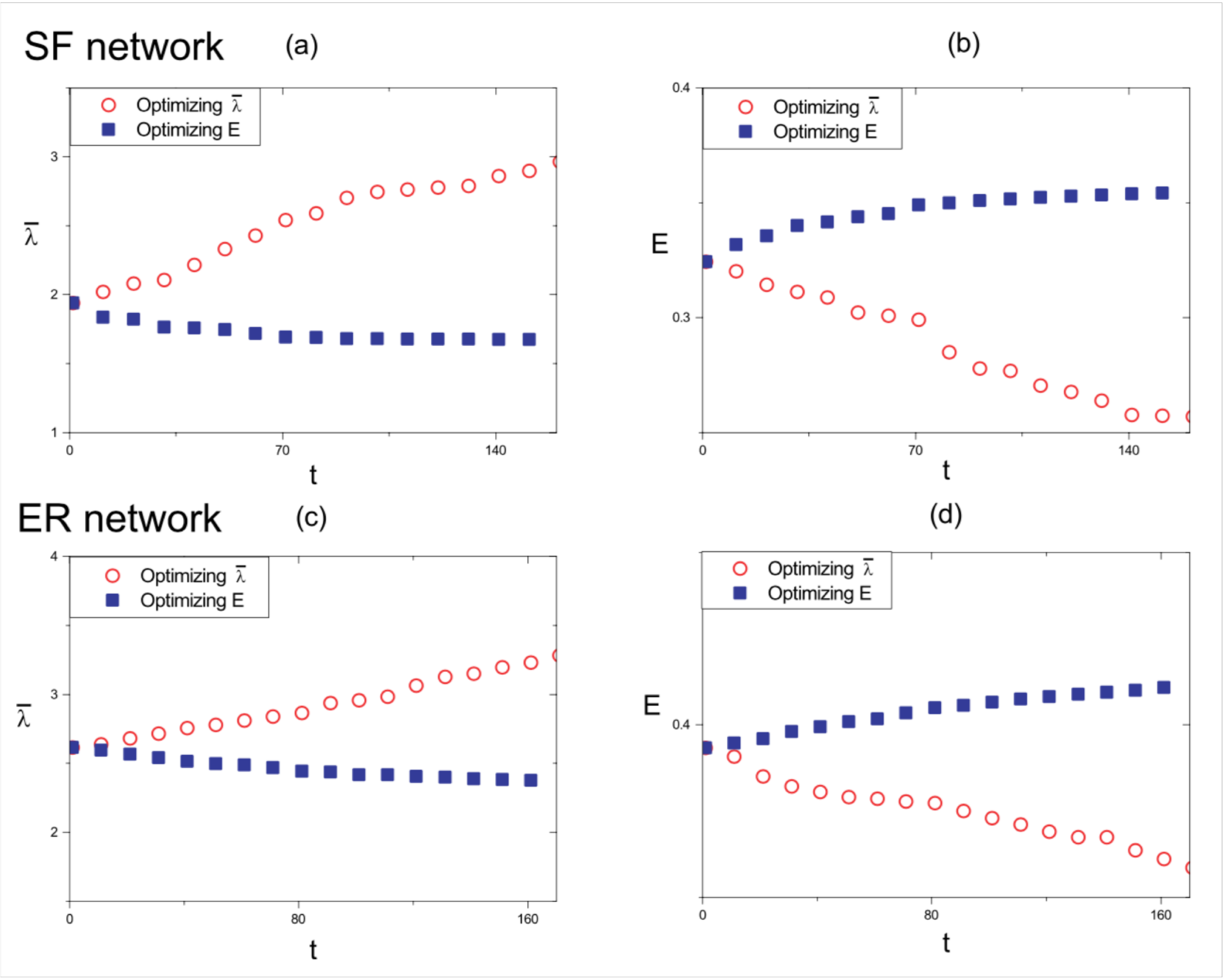
A single-objective optimization model

- Analyze the relation between robustness and small world effect optimizing them **separately**
- Degree-preserving greedy optimization algorithm
    - Degree conserved
    - Optimized network connected
- Rewring accepted if:
    - Objective improved
    - Network is connected



Degree-preserving rewiring process

A first hint



Natural connectivity $\bar{\lambda}$

Efficency E

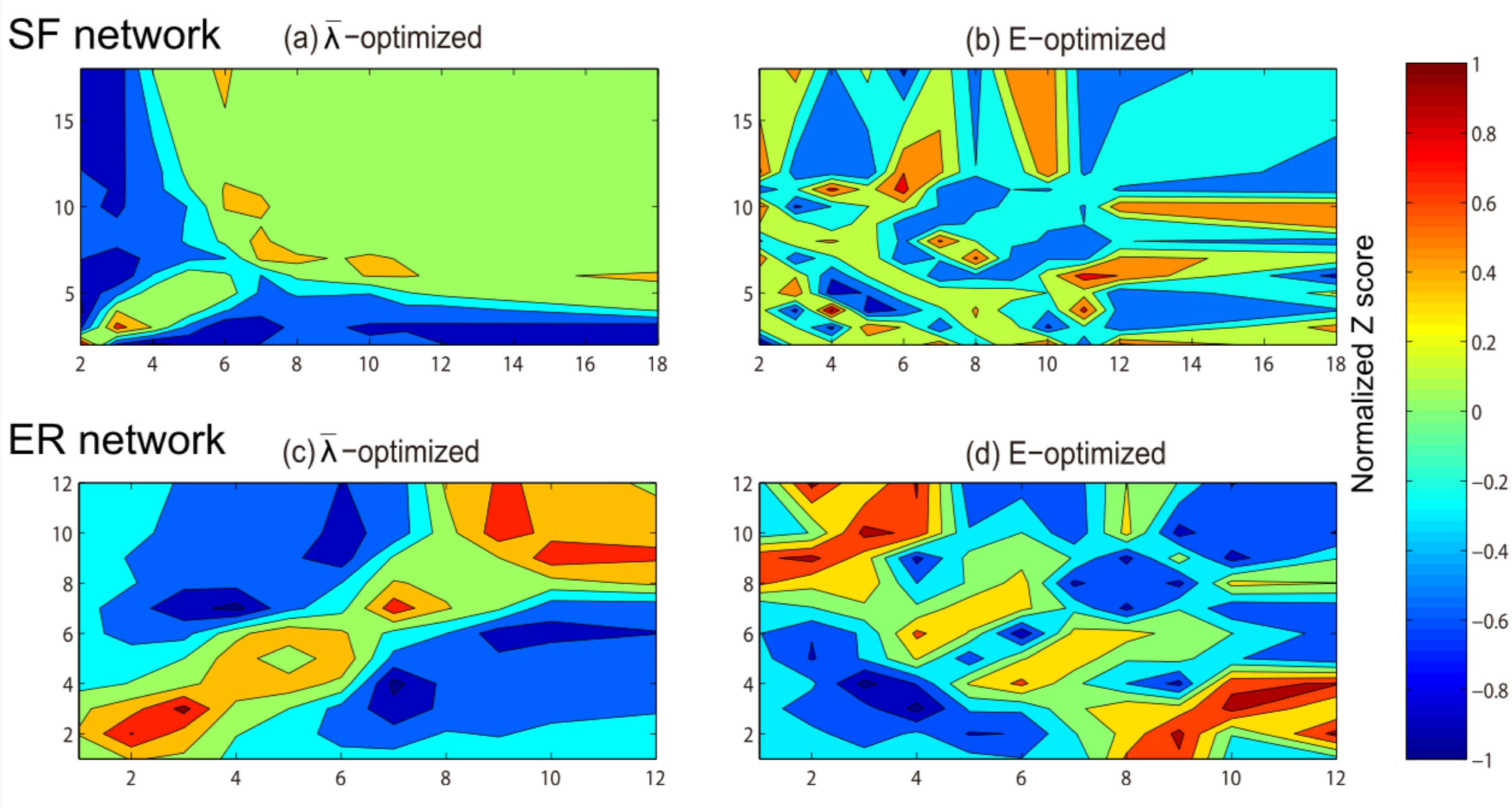
Degree correlation

- Significant property since the degree is fixed in optimization
- Statistical significance is described by the **Z score**, which reflects density of connections

$$Z(d_i, dj) = \frac{m(d_i, dj) - \langle m_r(d_i, dj) \rangle}{\sigma_r(d_i, dj)}$$

- $m(d_i, dj)$ is the number of links between nodes with degree d_i and nodes with degree d_j
- $\langle m_r(d_i, dj) \rangle$ and $\sigma_r(d_i, dj)$ are mean and standard deviation of $m(d_i, dj)$ in a randomized network sets generated from the specific network by executing degree-preserving rewiring algorithm

A second hint: correlation profiles

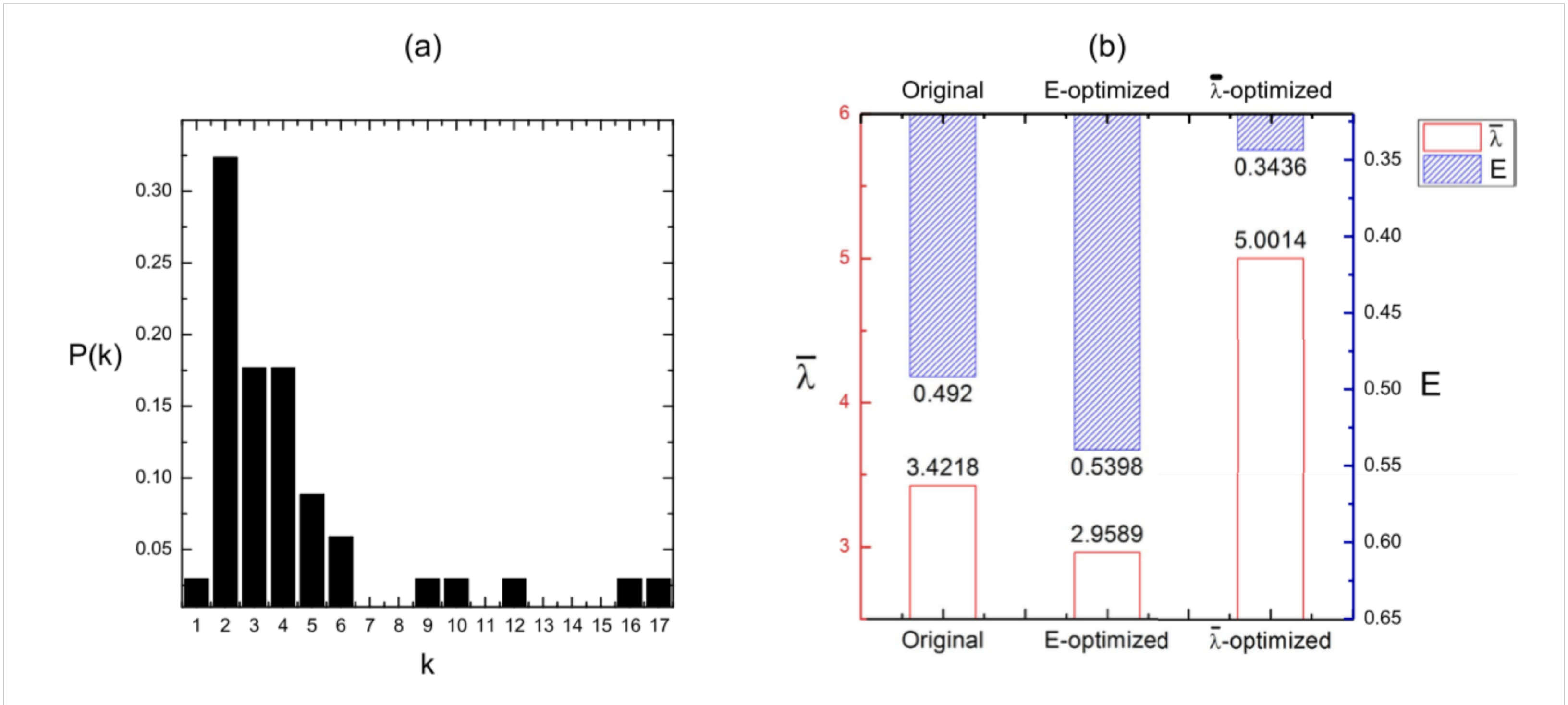


Assortative

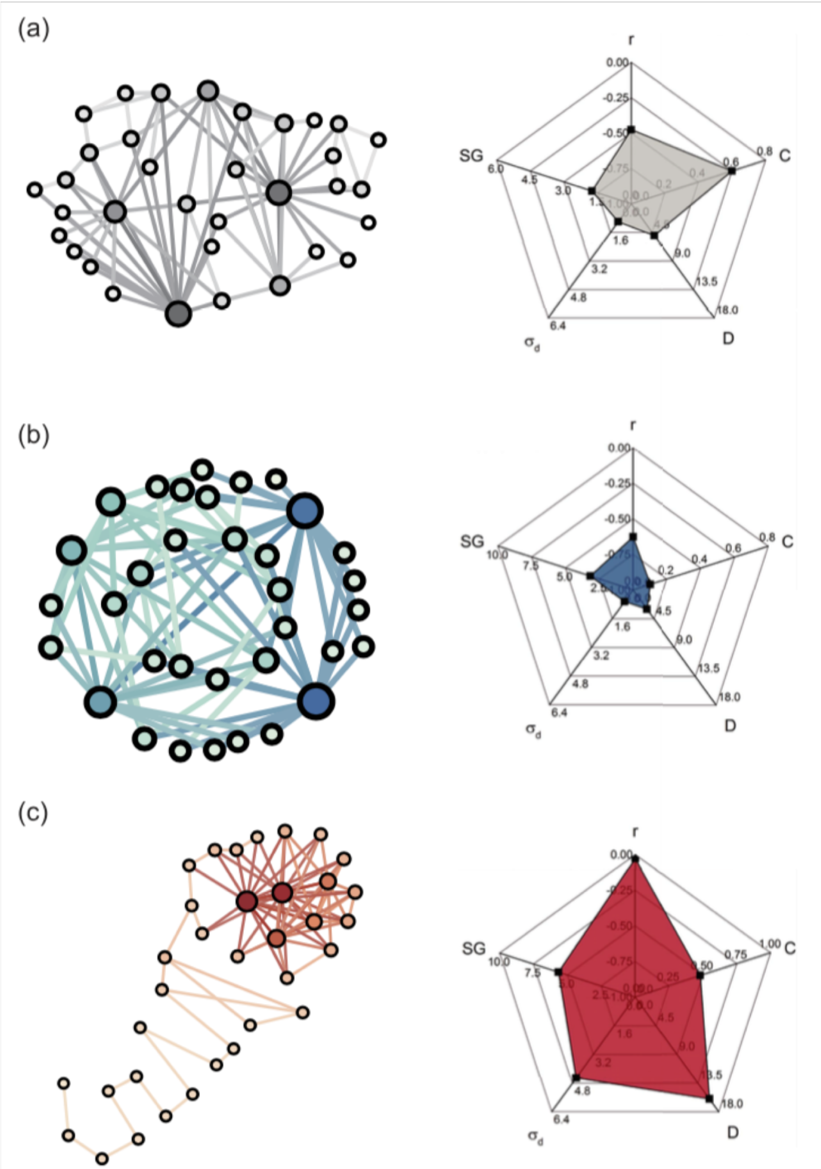
Disassortative

Optimization on a real network

- Zachary's karate club: a social network in a karate club at a US university in the 1970s
- 34 nodes
- 78 links



Different optimizations, different topologies



Original

***E*-optimized**
Multi-hub
Local star-like

$\bar{\lambda}$-optimized
Core-chain

- r = assortativity coefficient
- C = clustering coefficient
- D = network diameter
- σ_d = standard deviation of distance distribution
- SG = spectral gap

Multi-objective optimization model

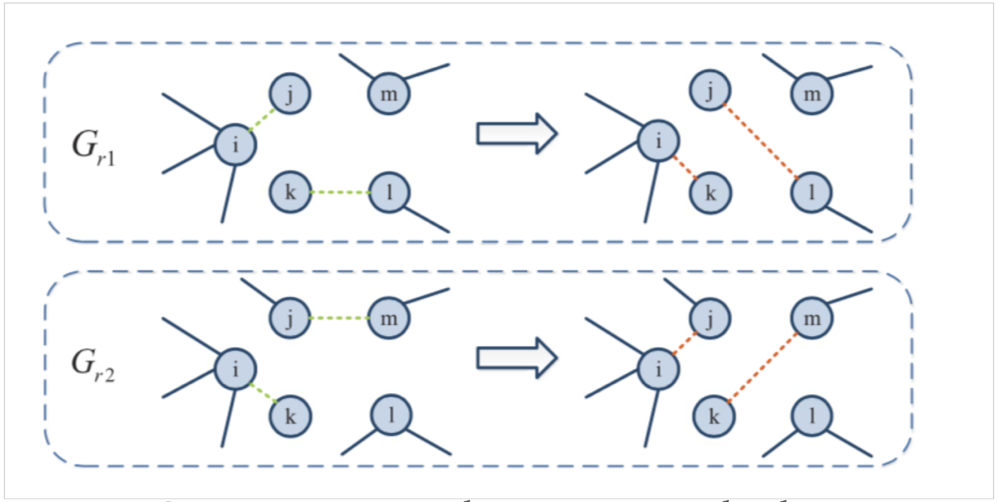
A tradeoff optimization model

- In order to consider both simultaneously robustness and small-world effect in the optimization, SMS-MOEA is employed
- **SMS-MOEA**: S-metric selection evolutionary multi-objective optimization algorithm
- MOEA Framework is a free and open source Java library which contains fast, reliable implementations of many state-of-the-art multi-objective evolutionary algorithms
- Used to obtain the Pareto-optimal front of $\overline{\lambda}$ and E, i.e. the best possible set of non-dominating solutions

SMS-MOEA (1/2)

- Part of the family of evolutionary algorithms, a generic population-based heuristic optimization algorithms which use mechanisms inspired by biological evolution

- ***Crossover operator***: fuses the genetic information from a pair of chromosomes and generate a new chromosome.

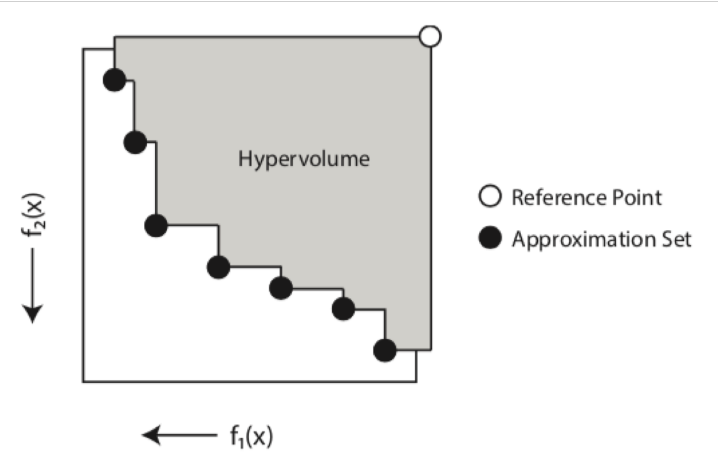


Crossover operation between two randomly selected networks G_{r1} and G_{r2}

SMS-MOEA (2/2)

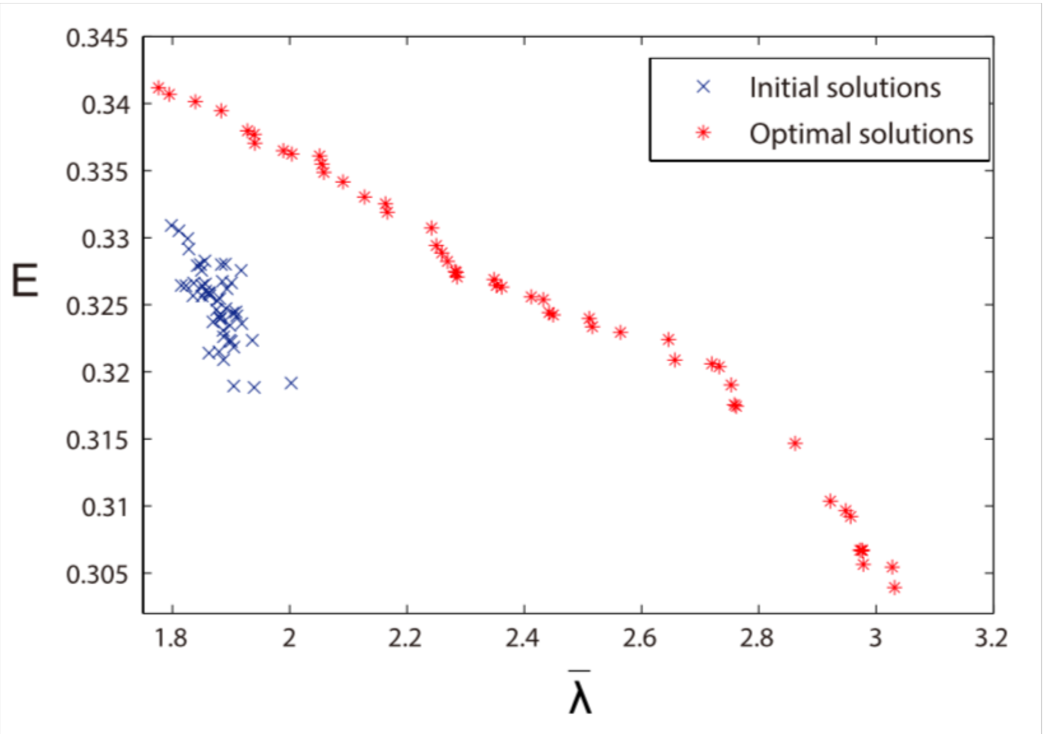
- ***Mutation operator***: aims to search new solutions in a local area to accelerate the convergence. Rewiring process as the mutation operation
- ***Reduce operator***: when a new network is added to the population, remove the inferior solution
  - SMS-MOEA maximizes the hypervolume of objectives
  - Hypervolume: area under Pareto-curves and bounded by reference point

$$\Delta\varphi(p_i, \mathbf{P}) = (\bar{\lambda}(p_{i+1}) - \bar{\lambda}(p_i))(E(p_{i+1}) - E(p_i))$$



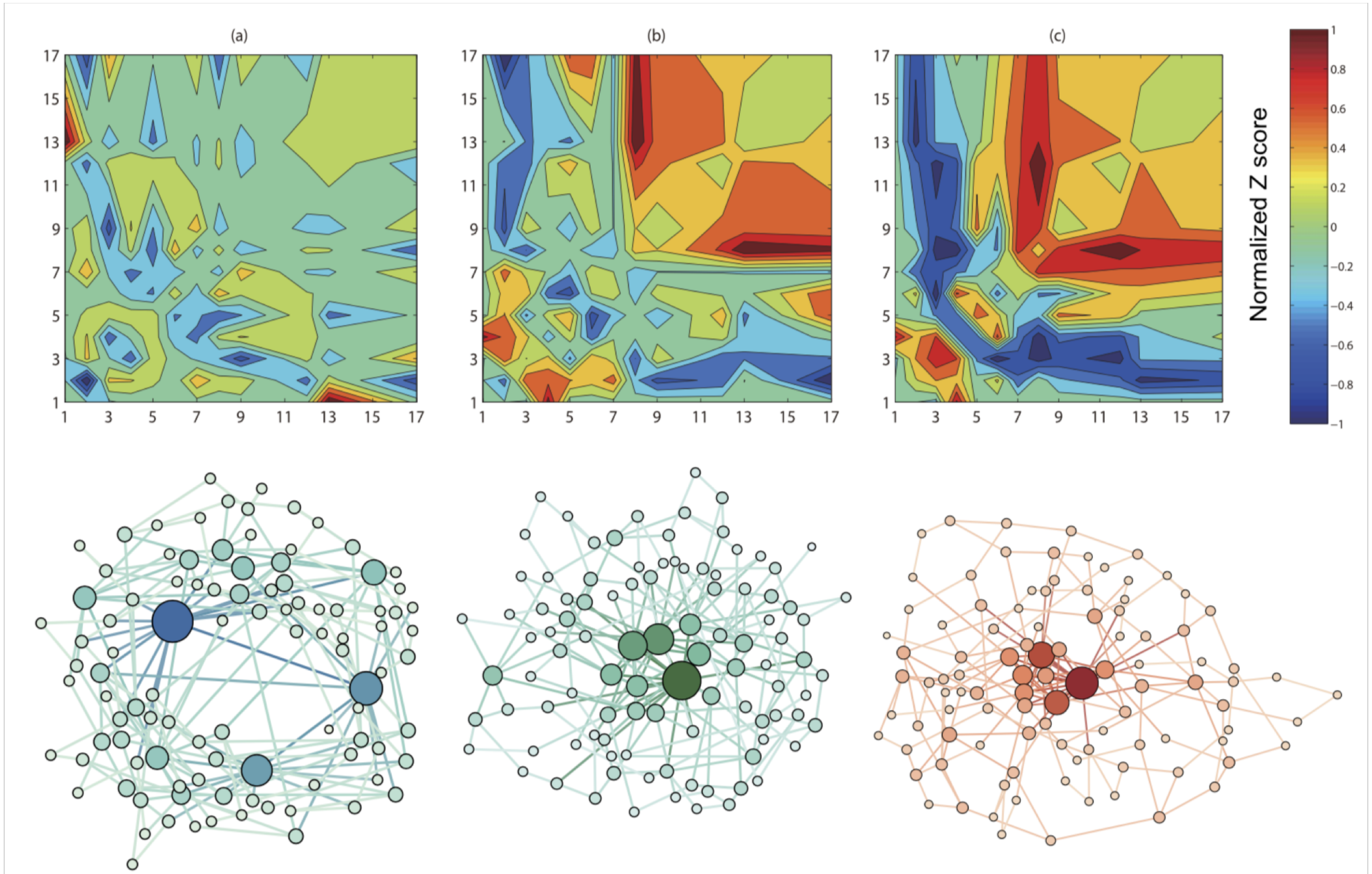
SMS-MOEA: parameters

- Population size = 50
- Crossover probability $P_c = 0.9$
- Mutation probability $P_m = 0.05$
- Initial solutions: generated from a SF network with $N = 100,\ \mathrm{L} = 100,\ \gamma = 3$ executing the mutation operator for 10^3 times



Pareto-optimal solutions set

- Visualization and correlation profiles
  a) High E, low $\overline{\lambda}$
  b) Both relatively high E and $\overline{\lambda}$
  c) High $\overline{\lambda}$, low E



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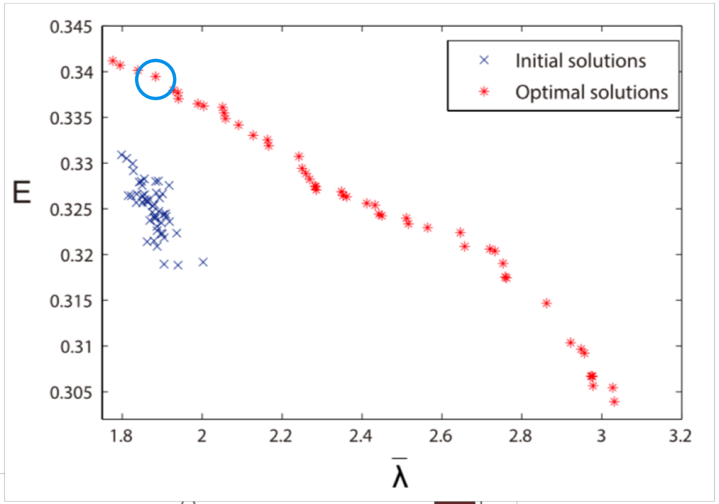

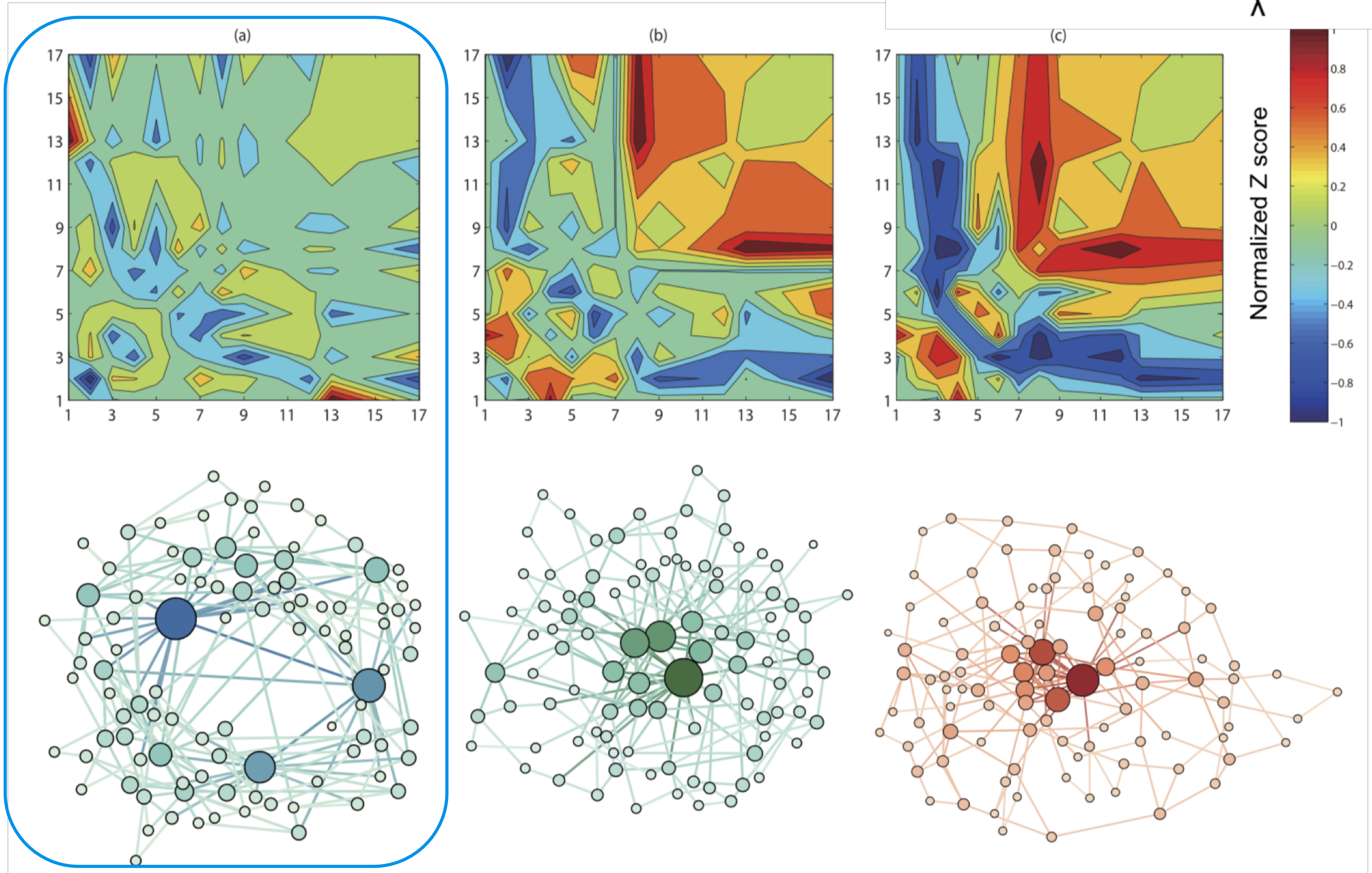

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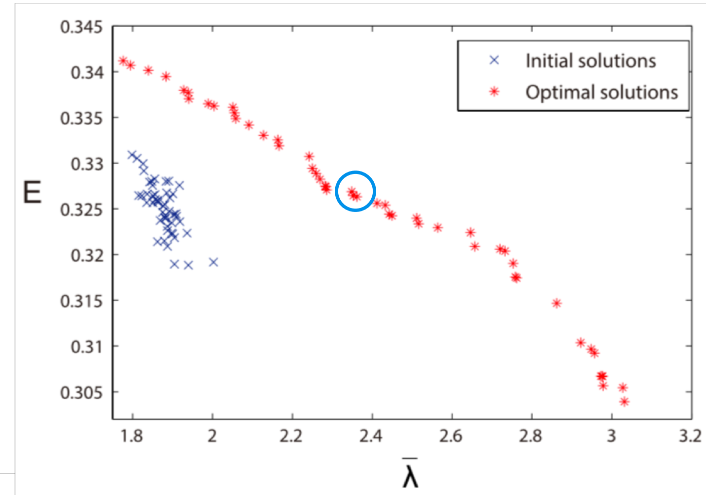

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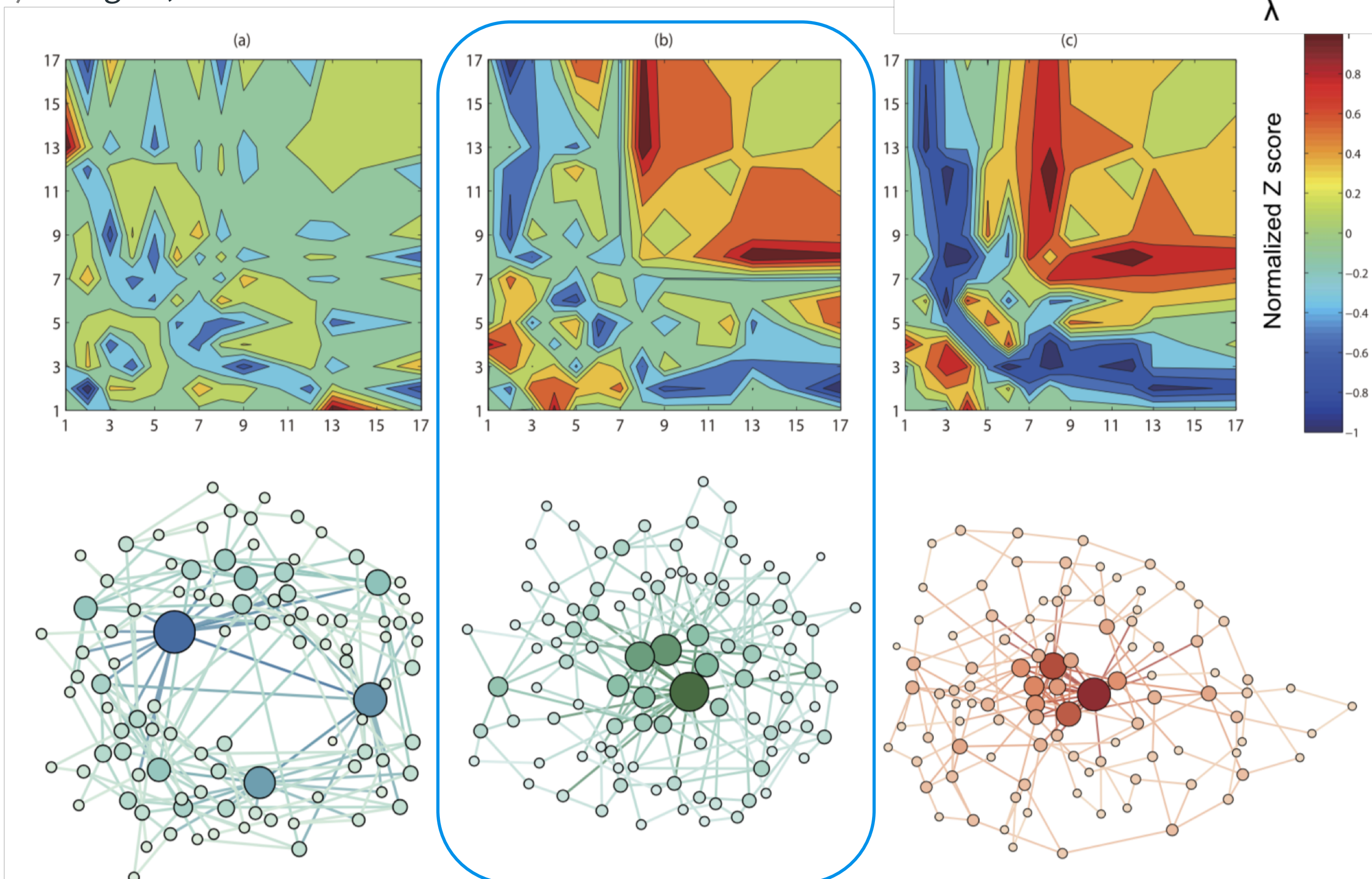

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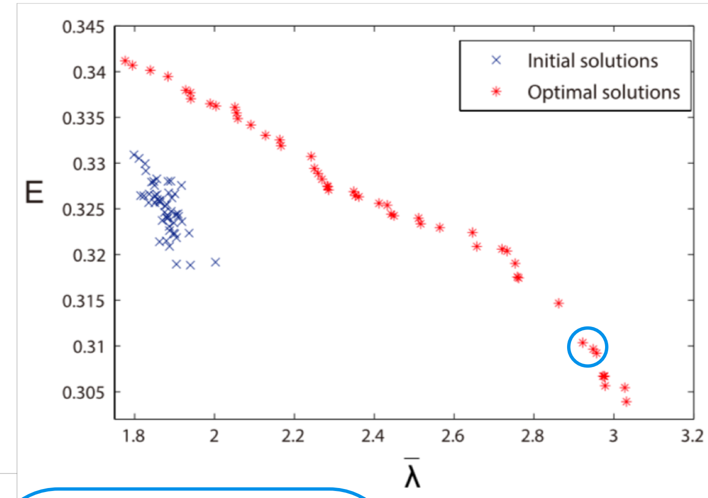
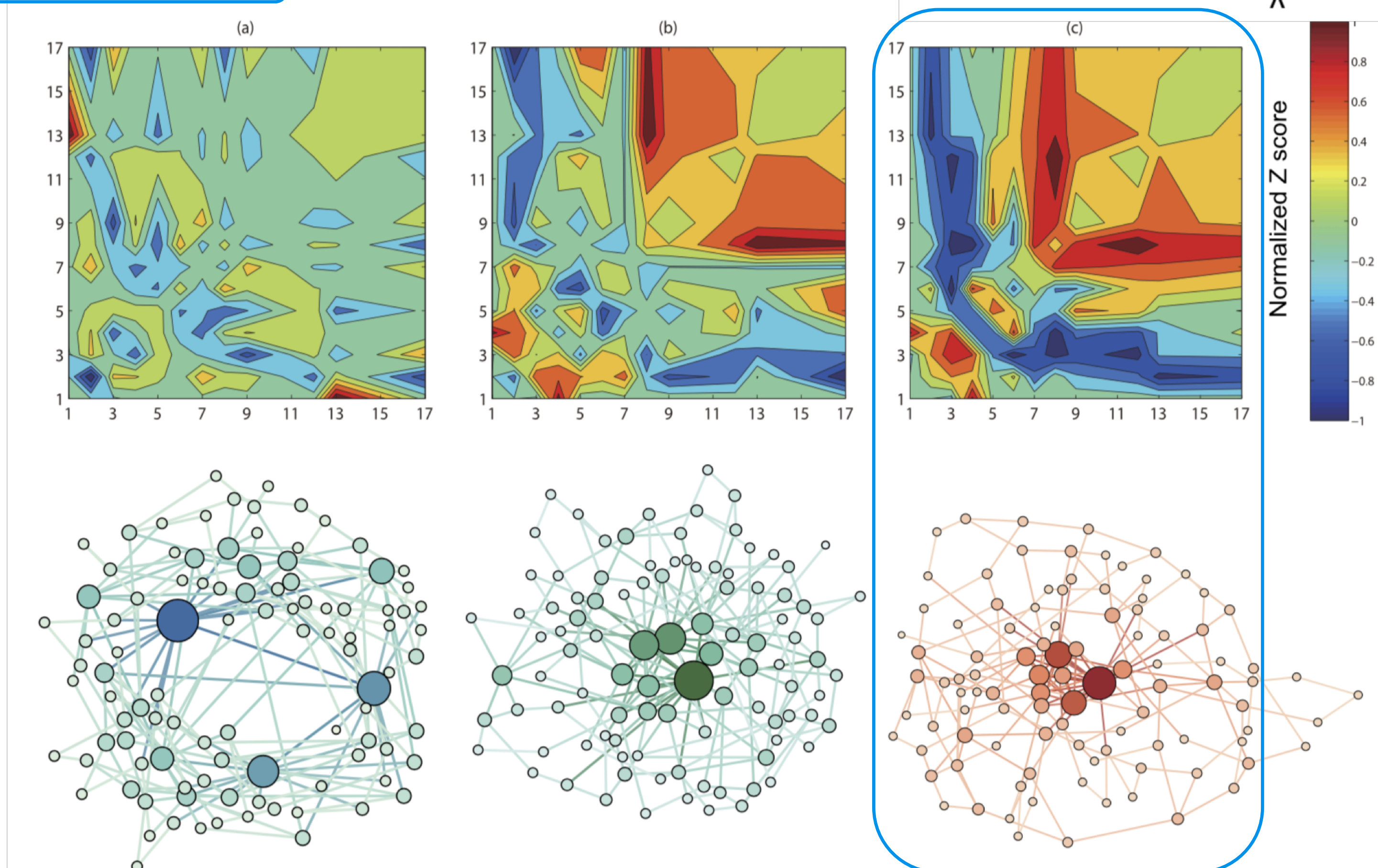




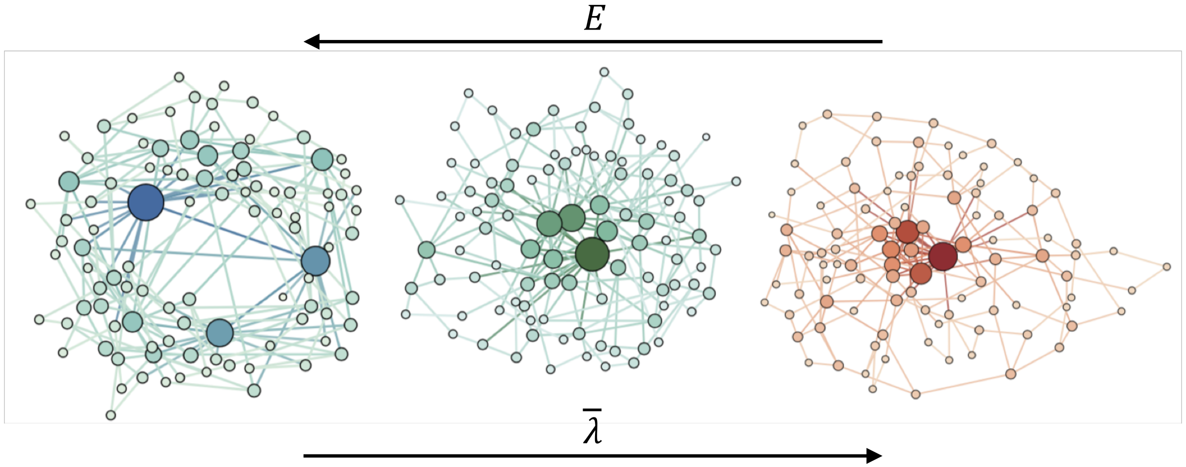
Conclusion

Discussion

- Robustness and small world effect are of great importance for designing and optimizing network topology
- They are in a conflicting relation in optimization while preserving the node degree
- A tradeoff using a multi-objective optimization model is possible

Network topologies

- Efficient network shows a **multi-hubs star-like** structure proved to be fragile for removals of high degree nodes
- Robust network has a **core-chain** topology.
  Long chain (ring) substructure has problems with communication
- Tradeoff network exhibit a **core-periphery** structure
  - Optimizing robustenss strenghtens core link density and expand periphery
  - Optimizing small-world effect weakens core and fragment periphery



Future works

- Take into account other constraints such as:
  - Geography
  - Rewiring limitations

- Investigate the tradeoff between robustness and small world effect in:
  - Directed networks
  - Weighted networks

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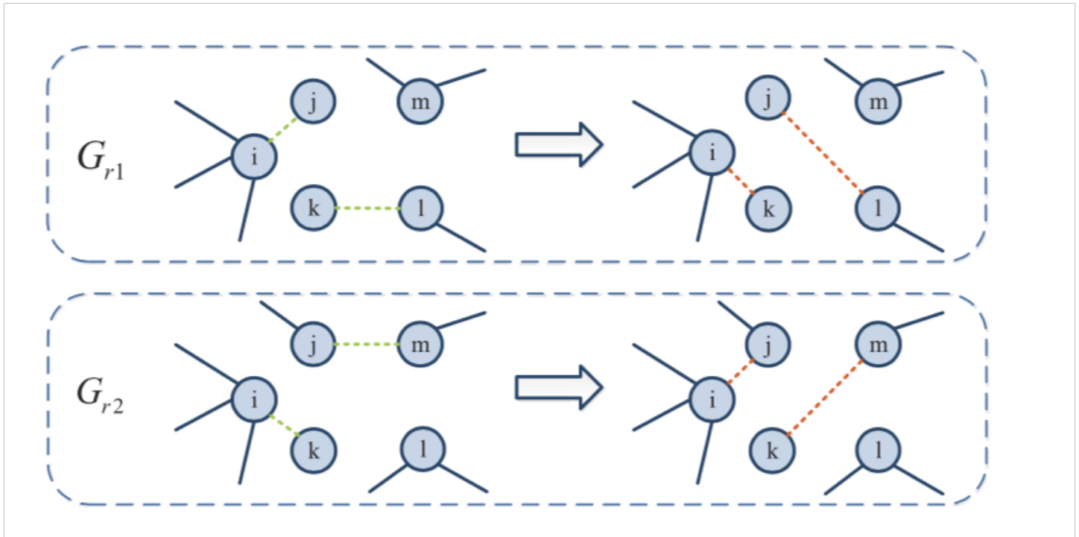
Thank you

Questions?

Crossover operator: a detailed explanation

- G_{r1} and G_{r2} randomly selected for the crossover operation
- $V_i(G_{r1})$ = set of neighbours of node i in G_{r1}
- $\overline{V}_i(G_{r1})$ = nodes connected to i in G_{r1} but disconnected in G_{r2}
- $\overline{V}_i(G_{r1})$ = j and $\overline{V}_i(G_{r2})$ = k
- Randomly select a node m in G_{r2} connected to j and disconnected to k
  - G_{r1}: links e_{ij} and e_{kl} are removed and links e_{ik} and e_{jl} are added
  - G_{r2}: links e_{ik} and e_{jm} are removed and links e_{ij} and e_{km} are added



Transition process of network topology

- Properties are in ascending order of their value of $\bar{\lambda}$